Fracture toughness of snow in shear under friction

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The fracture toughness of snow in shear is one of the most relevant parameters when studying the mechanics of snow slab avalanche release. Double-cantilever snow beams were loaded asymmetrically to determine K_{IIc}. If crack surfaces touch under applied pressure, the fracture toughness in shear of snow ($\rho = 247\ \mathrm{kg\,m^{-3}}$) is $K_{IIc} = 680 \pm 60\ \mathrm{Pa\,m^{1/2}}$. Screening of the stress intensity at the crack tip occurs by Newtonian friction with $k = 1.3 \pm 0.47$ along the crack faces.

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I. INTRODUCTION

Snow is ductile at low and brittle at high deformation rates [1,2]. Release of snow slab avalanches can occur by formation of shear cracks in the snow cover between two snow layers with different properties [3]. Once a failure at an interface between two snow layers has reached a critical size [4], the release process becomes fast, and linear elastic fracture mechanics, as familiar from industrial materials [5], applies. The relevant material parameter is the fracture toughness of snow in shear, K_{IIc}. Although this parameter is commonly used in models of snow slab avalanches [6], it has not been measured so far.

The fracture toughness of snow of density $\rho = 178\ \mathrm{kg\,m^{-3}}$, in mixed mode with both shear and tension present, was measured as $(K_{Ic}^2 + K_{IIc}^2)^{1/2} = 430 \pm 90\ \mathrm{Pa\,m^{1/2}}$ in cantilever beams [7]. Under mixed mode conditions, tension opens the crack, and friction across the crack faces does not affect the failure due to shear. In a snow slab avalanche situation, where shear cracks might develop below the slab parallel to the slope, the weight of the snow not only produces shear (mode II), but also exerts a closing pressure on the crack. No mode I is present; on the contrary, the shear crack faces are pressed against each other. The relevant parameter for this situation is, therefore, the fracture toughness in shear, K_{II}, in the presence of friction between the crack faces. One expects the shear toughness of the snow to be larger when the crack faces are pressed together than when they are not [8,9]. This quantity is needed in snow slab avalanche theories, but has never been measured. We have devised an experiment, and report here on the results.

II. GEOMETRY OF EXPERIMENTS

The geometry chosen for measuring K_{II} with the crack surfaces touching under load resembles the geometries chosen for measuring K_I [10] and for mixed mode fracture [7]. The snow beams are merely sliced horizontally rather than vertically [Fig. 1]. In absence of finite element calculations one uses beam theory to calculate energies, energy release rates, and stress intensity factors. In geometries similar to ours, where the beam approximation [11,12] was checked against finite element solutions [13,14], it proved to be very good. To find the stress intensity present at the tip of the crack in Fig. 1(a), consider first the line loads p applied at $x = L$, one to the top and one to the bottom half beam, as shown in Fig. 2(a).

The beam equations $E\,I\,d^2w(x)/dx^2 = M$ for the vertical deflection $w(x)$ have to be solved for the full beam and each of the two half beams. E is the Young's modulus. For the full beam the moment of inertia per unit thickness is $I = (2h)^3/12$ and the moment $M = 2p(L-x)$. For each of the half beams they are $I = h^3/12$ and $M = p(L-x)$. The two half beams bend the same way and touch each other without load transfer. For the full beam, the boundary condition is $w(0) = w'(0) = 0$; continuity requires that deflection and slope of the full and half beams match at $x = L - a$. One obtains

$$w(L) = \frac{p}{Eh^3}\ (L^3 + 3a^3). \tag{1}$$

If, in addition, a line load q is applied on the top, half of it is transmitted across the cut. In Eq. (1), p is replaced by $p + (q/2)$. The elastic energy stored in the whole arrangement is

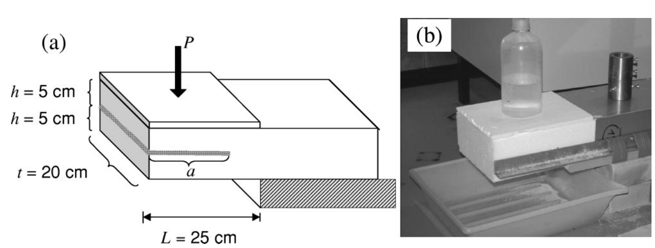

FIG. 1. Experimental setup. (a) Symmetrically sliced beam with externally applied weight P. Half the external load is transferred between the split beams and causes friction. Height $2h = 10$ cm, width $t = 20$ cm, cantilever length $L = 25$ cm, and critical crack length a_c is approximately 20 cm. (b) Photograph of the actual arrangement.

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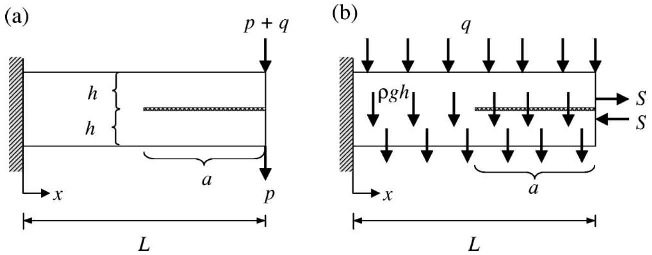

FIG. 2. Beam split over a length a. (a) Vertical line forces $p+q$ and p are applied to the top and the bottom of the half beams, respectively. (b) Experimental situation: Continuous loading $\rho gh+q$ and ρgh applied to the top and bottom, plus horizontal push-pulling forces S in the crack plane.

$$W=2\left(\frac{1}{2}\right)w(L)\left(p+\frac{q}{2}\right)=\left(p+\frac{q}{2}\right)^2\ \frac{L^3+3a^3}{Eh^3}. \qquad (2)$$

According to linear elastic fracture mechanics [5], the energy release rate dW/da is related to the stress intensity at the crack tip by

$$\frac{dW}{da}=\frac{K_{II}^2}{E}. \qquad (3)$$

The result for the line force loading considered is, therefore,

$$K_{II}=3\left(p+\frac{q}{2}\right)ah^{-3/2}. \qquad (4)$$

This result is strikingly simple; the loading enters in the form of the moment it creates in the plane of the crack front, i.e. $(p+q/2)a$. For the snow beam the body loading $p=\rho gh$ and the external loading q are continuously distributed, as shown in Fig. 2(b). Integration of Eq. (4) gives a positive stress intensity that tends to elongate the crack,

$$K_{II}^{+}=3\left(\rho gh+\frac{q}{2}\right)\frac{a^2}{2}h^{-3/2}. \qquad (5)$$

If, in addition to the vertical loadings, a horizontal loading S is present in the crack plane, the created tension or compression and the bending moments in the half beams contribute to the elastic energy W and its release rate dW/da and diminish the stress intensity by

$$K_{II}^{-}=S(1+3^{1/2})h^{-1/2}=2.73Sh^{-1/2}. \qquad (6)$$

Assuming that in our experiments Newtonian friction with a coefficient k prevails, it creates a shear stress $\tau_f=kq/2$ across the surfaces, being equivalent to forces $S=kqa/2$ acting in the crack plane. The total stress intensity is thus

$$K_{II}=K_{II}^{+}\ +K_{II}^{-}=3\left(\rho gh+\frac{q}{2}\right)\frac{a^2}{2}\ h^{-3/2}-2.73\ \frac{kqa}{2}\ h^{-1/2}. \qquad (7)$$

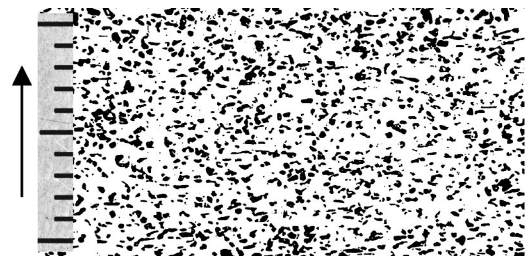

FIG. 3. Microstructure of snow type tested. Binarized picture of a surface section. Black denotes ice, white is the pore space ($\rho=250\ \mathrm{kg\,m^{-3}}$). Scale given is 1 cm. Arrow points to the snow surface (top margin was parallel to the snow surface).

The crack propagates catastrophically if the total stress intensity $K_{II}=K_{II}^{+}-K_{II}^{-}$ equals the fracture toughness K_{IIc} of the material. This happens at a critical crack length a_c. For the Newtonian friction, the left-hand side of Eq. (7), K_{IIc}, is a linear function of $(qa/2)h^{-1/2}$. By plotting the driving part K_{II}^{+} [Eq. (5)] as function of $(qa/2)h^{-1/2}$, for $a=a_c$, one should obtain a straight line with slope $2.73k$. The intercept is the true, unscreened toughness of snow in shear, a quantity independent of how the loading has been achieved. The apparent toughness is higher, because the friction across the crack surfaces screens a part of the loading.

III. RESULTS

Snow of density $\rho=247\pm11\ \mathrm{kg\,m^{-3}}$ was harvested at the Weissfluhjoch above Davos (Switzerland) in the form of beams (10 cm$\times$20 cm$\times$50 cm) and tested at $-10\,°\mathrm{C}$ in the cold laboratory of the Swiss Federal Institute for Snow and Avalanche Research SLF. At the time of testing, the snow type was characterized according to the ICSSG [15] as small rounded grains and partly decomposing and fragmented precipitation particles of 0.3–0.7 mm in size. Snow hardness index was estimated as 1–2. *In situ* temperature at the time of harvesting was $-9\,°\mathrm{C}$. In order to further characterize the snow texture, samples were preserved and subsequently analyzed with the surface section technique [Fig. 3].

The beams were cantilevered at 25 cm and loaded with weights $0<P<10$ N over the protruding surface 25 cm $\times$20 cm. This corresponds to $0<q<200$ Pa in the above equations. They were cut with a fishing line in the middle along the neutral axis, parallel to the snow layering, to a depth a_c, at which point the two half beams broke off vertically.

Figure 4 shows the appropriate plot for the assumption of Newtonian friction. Since Eq. (7) is valid only for pressured contact, $q>0$, slope and intercept are determined only for those points. The value of the unscreened toughness is 680 $\pm60\ \mathrm{Pa\,m^{1/2}}$, the slope gives a friction coefficient $k=1.3\pm0.47$. The mean values of the $q=0$ points, 630 $\pm90\ \mathrm{Pa\,m^{1/2}}$, is the toughness for a crack with surfaces touching without being externally loaded.

IV. DISCUSSION AND CONCLUSIONS

The crack surfaces can be in three states: (1) separated, (2) under forceless contact, and (3) at contact under pressure.

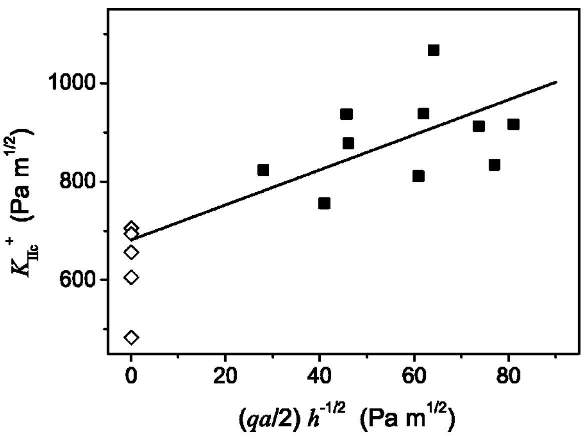

FIG. 4. The driving stress intensity K_{II}^{+} [Eq. (5)] as a function of $(qa/2)h^{-1/2}$, as appropriate for screening by Newtonian friction [Eq. (7)]. Full squares denote stress intensity factors from experiments with external loading ($q>0$) and open diamonds for the cases with $q=0$. The linear relation is statistically significant ($p=0.028$). The intercept is the unscreened toughness in shear, $K_{IIc}=680\pm60$ Pa m$^{1/2}$. The slope 3.56 corresponds to a friction coefficient $k=1.30\pm0.47$.

The respective toughness in shear is: (1) $K_{IIc}=430\pm90$ Pa m$^{1/2}$ if an opening mode I, however small, is present so that the crack surfaces do not touch [7]; (2) $K_{IIc}=630\pm90$ Pa m$^{1/2}$ if no opening mode I is present, the surfaces just touch but without being pressed against each other; and (3) $K_{IIc}=680\pm90$ Pa m$^{1/2}$, if the shear crack is closed under normal pressure. This is the quantity relevant for avalanche theories.

The medium situation, where the surfaces touch without pressure, is specific to the beam geometry. It is artificial and cannot occur in the avalanche context. Because of the grain structure of snow, and the imperfection of the cut, the surfaces have a certain roughness and cannot glide over each other without friction. This situation is qualitatively different from the pressurized situation, $q>0$, therefore the analysis according to Eq. (7) must exclude the $q=0$ points.

These results confirm the assumption that shear toughness with friction present is higher than that without friction [8,9]. Friction is Newtonian, with a coefficient $k=1.3\pm0.47$. This is in accordance with the measured friction coefficients for slowly sliding snow blocks [16]. Fracture occurs if the sum of loading and screening stress intensities exceeds the toughness, $K_{II}^{+}-K_{II}^{-}>K_{IIc}=680$ Pa m$^{1/2}$.

Only for the beam geometry, the expressions in Eq. (7) are appropriate for K_{II}^{+} and K_{II}^{-}. For other geometries, for example, a snow slab on a slope, loaded by its own weight, the appropriate expressions must be used. For K_{II}^{+} this is the stress intensity produced by the loading. For K_{II}^{-} it is the stress intensity caused by a shear stress $k=1.3$ times the pressure loading the crack faces. Friction always stabilizes the crack (at a given loading it increases the critical crack length, at a given crack length it increases the critical load). One concludes that friction decreases the risk of avalanches that occur by shear failure. However, even by including friction, the critical crack size under shear relevant for snow slab avalanche release remains small ($a_c<1$ m) as previously shown [7].

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